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Boundary estimates of $p$-harmonic functions
in a metric measure space

Hiroaki Aikawa
(Shimane University)

Let $(X, d, \mu)$ be a metric measure space equipped with a doubling measure supporting a $(1, p)$-Poincaré inequality for $1 < p < \infty$. The notion of $p$-harmonicity is defined on $X$ and a number of properties of classical harmonic functions have been extended to $p$-harmonic functions in $X$.

In this talk, we present two of such extensions. One is a Carleson type estimate for $p$-harmonic functions on a uniform domain or a John domain in $X$; the other is a characterization of domains in $X$ for which $p$-harmonic extensions of Hölder continuous boundary data are globally Hölder continuous. These results are new even if $X$ is the Euclidean space. This talk is based on the joint works with N. Shanmugalingam [1] and [2].

REFERENCES

Equivalent norms on weighted Lipschitz spaces on the unit ball

Hong Rae Cho
(Pusan National University)

A continuous function $\omega : (0, \infty) \to (0, \infty)$ with $\limsup_{t \to 0^+} \omega(t) = 0$ will be called a majorant if $\omega(t)$ is increasing and $\omega(t)/t$ is nonincreasing for $t > 0$. If, in addition, there is a constant $C(\omega) > 0$ such that

$$\int_0^\delta \frac{\omega(t)}{t} \, dt + \delta \int_\delta^\infty \frac{\omega(t)}{t^2} \, dt \leq C(\omega) \cdot \omega(\delta).$$

whenever $0 < \delta < 1$, then we say $\omega$ is a regular majorant.

For $E \subset \mathbb{R}^N$, the Lipschitz-type space $\Lambda_\omega(E) = \{ f : \|f\|_{A_\omega(E)} < \infty \}$ consists of the functions $f : E \to \mathbb{R}^N$ satisfying

$$\|f\|_{A_\omega(E)} = \sup_{E} |f| + \sup \left\{ \frac{|f(z_1) - f(z_2)|}{\omega(|z_1 - z_2|)} : z_1, z_2 \in E, z_1 \neq z_2 \right\} < \infty.$$

Let $B$ be the unit ball in $\mathbb{C}^n$. In this chapter we introduce several equivalent norms on $\Lambda_\omega(B)$, each of them depending only on the modulus of the function in question.
Certain operators on Bergman spaces on strongly pseudoconvex domains that improve integrability

Željko Ćučković
(University of Toledo)

We study two types of operators that improve integrability. First, we characterize bounded and compact weighted composition operators acting on Bergman spaces on strongly pseudoconvex domains in $\mathbb{C}^n$. We also give estimates on their essential norms (joint work with Ruhan Zhao). Then we study the mapping properties of Toeplitz operators on $L^p$ on strongly pseudoconvex domains whose symbol is a positive power of the distance to the boundary (joint work with Jeff McNeal).
This talk mainly concerns homogeneous submodules of essentially normal $\mathcal{U}$-invariant Hilbert modules. When a homogeneous submodule is essentially normal, its spectrum, essential spectrum are completely described by zero variety of the submodule. In dimensions $d = 2, 3$, it is shown that the $C^*$-extension determined by the corresponding quotient module is not trivial if a homogeneous submodule is essentially normal. In dimension $d = 2$, and in the case of finite multiplicity, it is proved that each homogeneous submodule is $p$-essentially normal for $p > 2$. The talk will also gives an explicit expression for $K$-homology invariant defined in the case of dimension $d = 2$. 

$\mathcal{U}$-invariant Hilbert modules and $K$-homology

Kunyu Guo

(Fudan University)
Minimum moduli of weighted composition operators on algebras of analytic functions

Takuya Hosokawa
(Nippon Institute of Technology)

We study the minimum moduli of weighted composition operators on the disk algebra and the space of bounded analytic functions.
TBA

Junyun Hu
(Jiaxing University)

TBA
Rank one commutators on invariant subspaces of the Hardy space on the bidisk

Keiji Izuchi* and Kou Hei Izuchi
(Niigata University)

A closed subspace \( M \) of \( L^2(\Gamma^2) \) is called invariant if \( L_z M \subset M \) and \( L_w M \subset M \). For an invariant subspace \( M \) of \( L^2(\Gamma^2) \), we have two natural operators \( R_z = P_M L_z \) and \( R_w = P_M L_w \) on \( M \). Mandrekar showed that for \( M \) of \( H^2(\Gamma^2) \), \( [R_w, R_z^*] = 0 \) if and only if \( M = \varphi H^2(\Gamma^2) \) for an inner function \( \varphi \). So, this condition characterizes Beurling type invariant subspaces in \( H^2(\Gamma^2) \). Nakazi posed the following conjecture: if \( [R_w, R_z^*] = [R_w, R_z^*]^* \), then \( [R_w, R_z^*] = 0 \). Ohno and the first author showed the following:

Let \( M \) be an invariant subspace of \( L^2(\Gamma^2) \). Then the following conditions are equivalent.

(i) \( [R_w, R_z^*] = [R_w, R_z^*]^* \) and \( [R_w, R_z^*] \neq 0 \).

(ii) \( M = \varphi \left( H^2(\Gamma^2) \oplus \left( \sum_{j=0}^{\infty} \oplus \mathbb{C} \cdot z^j \frac{\bar{w}}{1 - rz\bar{w}} \right) \right) \),

where \( \varphi \) is a unimodular function on \( \Gamma^2 \) and \( r \) is a real number with \( 0 < |r| < 1 \).

After that, Nakazi asked: is there an invariant subspace \( M \) in \( H^2(\Gamma^2) \) having the form in (ii)? We answer his question affirmatively. It is not so difficult to see that \( \text{rank} \ [R_w, R_z^*] = 1 \) for an invariant subspace \( M \) of the form in (ii). Until now, there are not so many studies on invariant subspaces of \( H^2(\Gamma^2) \) with \( \text{rank} \ [R_w, R_z^*] = 1 \). We are interested in such invariant subspaces, and discuss on this subject. Yang essentially showed that \( \text{rank} \ [R_w, R_z^*] = 1 \) for invariant subspaces of the form:

\[
\varphi \left( q_1(z) H^2(\Gamma^2) + q_2(w) H^2(\Gamma^2) \right),
\]

where \( \varphi \) is inner, and \( q_1(z), q_2(w) \) are one variable inner.

Let

\[
M_1 = \varphi \left( H^2(\Gamma^2) \oplus \left( \sum_{j=0}^{\infty} \oplus z^j \text{span} \left\{ \frac{\bar{w}^i}{1 - az\bar{w}^n}; 1 \leq i \leq n \right\} \right) \right).
\]

**Theorem.** Let \( M_1 \) be an invariant subspace of \( H^2(\Gamma^2) \) given in (1). Then \( \text{rank} \ [R_w, R_z^*] = 1 \) for \( M_1 \).
Equivalence between canonical domains for doubly connected planar domains

Moonja Jeong* (University of Suwon)
Jong-Won Oh (Yonsei University)
Masahiko Taniguchi (Kyoto University)

S. Bell in [1] made a conjecture on a new canonical domain of the non-degenerate \( n \)-connected planar domains and his conjecture was solved in [2]. The coefficient body of a new canonical domain for \( n = 2 \) was studied in [3].

Now let

\[
A(r) = \{ z \in \mathbb{C} : |z + 1/z| < r \}, \quad r > 2.
\]

The annulus and the above domain are canonical domains for \( n = 2 \) and we show the equivalence of them in an explicit formula. It is done by using the property of the Ahlfors map in [4] and [5]. Also we express it in another way by using the Teichmüller domain and the theta constants, and derive some identity.

REFERENCES

Another look at average formulas of Nevanlinna counting functions of holomorphic self-maps of the unit disk

Hong Oh Kim
(KAIST)

For a holomorphic self-map $\varphi$ of the unit disk $D$ on the complex plane, the Nevanlinna counting function $N_\varphi$ is defined by

$$N_\varphi(w) = \begin{cases} 
\sum_{\varphi(z)=w} \log \frac{1}{|z|}, & \text{if } w \in \varphi(U) \\
0, & \text{if } w \notin \varphi(U)
\end{cases}$$

It plays a very important role in the holomorphic change of variables by $w = \varphi(z)$ in the integral representation and in the study of the composition operator $C_\varphi(f) = f \circ \varphi$. The averages formulas $N_\varphi(w)$ around a circle and a disk are given and exploited to the explicit representation of the Nevanlinna counting functions of Rudin’s orthogonal functions. We also add another application of the average formulas to characterize a special class of inner functions.
Norm estimates for the Alexander transforms of convex functions of order alpha

Yong Chan Kim
(Yeungnam University)

The hyperbolic sup norm of the pre-Schwarzian derivative of a locally univalent function on the unit disk measures the deviation of the function from similarities. We present sharp norm estimates of the pre-Schwarzian derivatives for subclasses of univalent functions. We also consider the Alexander transforms in connection with pre-Schwarzian derivatives.
Zero products of Toeplitz operators with harmonic symbols

Boorim Choe and Hyungwoon Koo*
(Korea University)

On the Bergman space of the unit ball in $\mathbb{C}^n$, we solve the zero-product problem for two Toeplitz operators with harmonic symbols that have local continuous extension property up to the boundary. In the case where symbols have additional Lipschitz continuity up to the boundary, we solve the zero-product problem for multiple products with the number of factors depending on the dimension $n$ of the underlying space; the number of factors is $n + 3$. We also prove a local version of this result but with loss of a factor.
On a generalized Bergman projection

Ern Gun Kwon
(Andong National University)

For $-1 < \alpha < \infty$ and for a holomorphic self map $\varphi$ of the unit disc $D$, we consider a generalized Bergman projection $P_{\varphi, \alpha}$ defined by

$$P_{\varphi, \alpha}f(z) = \int_D \frac{f(w) \, dA_\alpha(w)}{(1 - \overline{w}\varphi(z))^{2+\alpha}}, \quad z \in D.$$  

The boundedness of the projection from $L^\infty(D)$ into Hardy families are expressed in terms of the growth rates of $\varphi$. 
Generalized interpolation in Hardy spaces

Daniel H. Luecking
(University of Arkansas)

The usual definition of interpolating sequences requires a particular definition of a norm on the sequence space being interpolated. While this norm is a natural one, I will discuss another one that seems to be just as natural. One characterization of interpolation sequences for $H^p$ is that they must be uniformly discrete in the hyperbolic metric and a certain discrete measure associated to the sequence is a Carleson measure. With the new norm, the characterization is almost the same: the measure must still be a Carleson measure, but the sequence need not be uniformly discrete. Equivalently, the sequence must be a finite union of the usual interpolating sequences.

If time permits, I will show how this relates to other characterizations of such finite unions, and to corresponding ideas in Bergman spaces.
Existence of tangential limits for $\alpha$-harmonic functions on half spaces

Yoshihiro Mizuta
(Hiroshima University)

Riesz [3] defined the notion of $\alpha$-harmonic functions on a domain $\Omega$ in the $n$-dimensional Euclidean space $\mathbb{R}^n$, as solutions of the fractional Laplace operators. We know that the Riesz potential of order $\alpha$, $0 < \alpha \leq 2$,

$$U_\alpha \mu(x) = \int |x - y|^{\alpha - n} d\mu(y)$$

for a nonnegative measure $\mu$ on $\mathbb{R}^n$ is $\alpha$-superharmonic in $\mathbb{R}^n$ and $\alpha$-harmonic outside the support of $\mu$; for this, see Riesz [3] and Landkof [2].

In the half space $H = \{x = (x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : x_n > 0\}$, consider

$$P_\alpha f(x) = c_\alpha \int_{\mathbb{R}^n \setminus H} \left( \frac{x_n}{|y_n|} \right)^{\alpha/2} |x - y|^{-n} f(y) dy$$

for a measurable function $f$ on $\mathbb{R}^n$, where $0 < \alpha < 2$ and $c_\alpha = \Gamma(n/2) \pi^{-n/2-1} \sin(\pi \alpha/2)$. Then it is seen that $P_\alpha f$ is $\alpha$-harmonic in $H$. Recently, Bass and You [1] have shown the existence of nontangential limits for $P_\alpha f$ with $f \in \Lambda^{p, \infty}_\beta(\mathbb{R}^n)$, which is the space of $L^p$ Hölder continuous functions of order $\beta$.

Our aim in this talk is to prove the existence of tangential limits for $P_\alpha f$, as an improvement of their result.

REFERENCES


Schatten-Herz type Toeplitz operators on the harmonic Bergman space

Kyunguk Na
(Korea University)

Motivated by a recent work of Loaiza et al. [9] for the holomorphic case on the disk, we introduce and study the notion of Schatten-Herz type Toeplitz operators acting on the harmonic Bergman space of the ball. We obtain characterizations of positive Toeplitz operators of Schatten-Herz type in terms of averaging functions and Berezin transforms of symbol functions. Our characterization in terms of Berezin transforms settles a question posed by Loaiza et al.

This is a joint work with Boorim Choe and Hyungwoon Koo.

REFERENCES

Compact operator in the Toeplitz algebra

Kyesook Nam
(Hanshin University)

$m$-Berezin transforms are introduced for bounded operators on the Bergman space of the polydisk. We show several properties of $m$-Berezin transform and use them to show that a radial operator in the Toeplitz algebra is compact iff its Berezin transform vanishes on the boundary of the polydisk.

This is a joint work with Dechao Zheng.
Some integration operators on analytic functions
in the unit disc

Takahiko Nakazi
(Hokkaido University)

Let \( \varphi \) be an analytic function on the open unit disc \( D \) in the complex plane. Put

\[
(M_{\varphi}f)(z) = \varphi(z)f(z),
\]
\[
(I_{\varphi}f)(z) = \int_{0}^{z} f'(\zeta)\varphi(\zeta)d\zeta
\]
and

\[
(J_{\varphi}f)(z) = \int_{0}^{z} f(\zeta)\varphi'(\zeta)d\zeta
\]
for a holomorphic function \( f \) on \( D \). In this lecture we study these operators on Hilbert spaces, for example, weighted Hardy spaces and weighted Dirichlet spaces. In particular, we study when \( M_{\varphi}, I_{\varphi} \) or \( J_{\varphi} \) is a Fredholm operator on a weighted Dirichlet space.
Products of composition and differentiation between Hardy spaces

Shûichi Ohno
(Nippon Institute of Technology)

We will discuss boundedness and compactness of the products of composition and differentiation between Hardy spaces.

REFERENCES

Characterization of the weighted Bergman spaces in terms of derivatives

Jong-Do Park
(Sogang University)

Let $\mathbb{B}_n$ be the unit ball in $\mathbb{C}^n$ and $d\nu_s$ is defined by $d\nu_s(z) = (1 - |z|^2)^s d\nu(z)$, where $d\nu$ is the normalized Lebesgue measure on $\mathbb{B}_n$. We study some properties of the operator $A_{l,s} : L^2_a(\mathbb{B}_n, d\nu_s) \to \mathbb{R}$ defined by

$$A_{l,s}(f) = \sum_{|J|=l} \|(1 - |z|^2)^l D^J f(z)\|^2_{L^2_a(\mathbb{B}_n, d\nu_s)},$$

where $J$ is a multi-index with length $l \geq 0$. The following is the basic formula. For any $m \geq 0$, we have

$$\sum_{|J|=l} \int_{\mathbb{B}_n} (1 - |z|^2)^m |D^J z^k|^2 d\nu(z) = \frac{k!n!\Gamma(m + 1)}{\Gamma(|k| + m + n - l + 1)} P(|k|, l),$$

where $P(q, r) := r!\binom{q}{r}$. Using this formula, we give an elementary proof of the fact that for any $s > -1$, $A_{0,s}(f)$ is finite if and only if $A_{N,s}(f)$ is finite for any positive integer $N$.

We also construct the inner product formula for the Bergman space in the unit ball $\mathbb{B}_n$: the inner product of two holomorphic functions can be represented as the sum of different weighted inner products of derivatives. Using this formula, we study bounded Toeplitz products on the Bergman space in the unit ball $\mathbb{B}_n$ in $\mathbb{C}^n$. 
Brennan’s conjecture for weighted composition operators

Wayne Smith
(University of Hawaii)

Brennan’s conjecture concerns integrability of the derivative of a conformal map \( \tau \) of the unit disk \( \mathbb{D} \). The conjecture is that, for all such \( \tau \),

\[
\int_{\mathbb{D}} (1/|\tau'|)^p dA < \infty
\]

holds for \(-2/3 < p < 2\). This is known for \(-2/3 < p \leq 1.421\).

We show Brennan’s conjecture is equivalent to a statement about weighted composition operators. Let \( \tau \) be as above and let \( \varphi \) be an analytic self-map of \( \mathbb{D} \). Define, for \( f \) analytic on \( \mathbb{D} \),

\[
(A_{\varphi,p} f)(z) = \left( \frac{\tau'((\varphi(z)))}{\tau'(z)} \right)^p f(\varphi(z)).
\]

There are always choices of \( \varphi \) that make \( A_{\varphi,p} \) bounded on the Bergman space \( L^2_a(\mathbb{D}) \). We are interested in the set of \( p \) for which there is a choice of \( \varphi \) (depending on \( \tau \)) that makes \( A_{\varphi,p} \) compact on \( L^2_a(\mathbb{D}) \). We show this happens if and only if \((1/\tau')^p \in L^2_a(\mathbb{D})\). Thus Brennan’s conjecture is equivalent to such a choice of \( \varphi \) existing for the range \(-1/3 < p < 1\), and this is known for \(-1/3 < p \leq .7105\).
Dual Toeplitz operators

Karel Stroethoff
(University of Montana)

A dual Toeplitz operator is defined to be multiplication followed by projection onto the orthogonal complement of a Bergman space in the space of square integrable functions on the domain. We will discuss properties of dual Toeplitz operators.
Invariant differential operators and their applications

Toshiyuki Sugawa
(Hiroshima University)

Various invariant differential operators have been studied by many authors. Particularly important are differential operators associated with spherical, Euclidean and hyperbolic metrics. According to the cases $\varepsilon = +1, 0, -1$, those metrics are described by $\lambda_\varepsilon(z)|dz| = |dz|/(1 + \varepsilon|z|^2)$ on $C_\varepsilon$, where $C_{\varepsilon+1} = \hat{\mathbb{C}}$ (the Riemann sphere), $C_0 = \mathbb{C}$ (the complex plane) and $C_{\varepsilon-1} = \mathbb{D} = \{|z| < 1\}$ (the unit disk). For example, given a holomorphic function $f : \mathbb{D} = C_{\varepsilon-1} \to C_\varepsilon$, the invariant derivatives associated with $(\lambda_{\varepsilon-1}, \lambda_\varepsilon)$ up to the third order are given by

$D_1f(z) = \frac{(1 - |z|^2)f'(z)}{1 + \varepsilon|f(z)|^2},$

$D_2f(z) = \frac{(1 - |z|^2)^2f''(z)}{1 + \varepsilon|f(z)|^2} - \frac{2\bar{z}(1 - |z|^2)f'(z)}{1 + \varepsilon|f(z)|^2} - \frac{2(1 - |z|^2)^2\overline{f(z)}f'(z)^2}{(1 + \varepsilon|f(z)|^2)^2},$

$D_3f(z) = \frac{(1 - |z|^2)^3f'''(z)}{1 + \varepsilon|f(z)|^2} - \frac{6(1 - |z|^2)^3\overline{f(z)}f'(z)f''(z)}{(1 + \varepsilon|f(z)|^2)^2} - 6\bar{z}(1 - |z|^2)^2f''(z) - \frac{6\bar{z}^2(1 - |z|^2)f'(z)}{1 + \varepsilon|f(z)|^2} + \frac{6\bar{z}(1 - |z|^2)^2\overline{f(z)}f'(z)^2}{(1 + \varepsilon|f(z)|^2)^2} + \frac{12\bar{z}(1 - |z|^2)^2\overline{f(z)}f'(z)^2}{(1 + \varepsilon|f(z)|^2)^2} + \frac{6(1 - |z|^2)^3\overline{f(z)}^2f'(z)^3}{(1 + \varepsilon|f(z)|^2)^3}.$

In the present talk, we recall the definition of these operators of an arbitrary order and then give some nontrivial relations between them. Finally, as applications, we give norm estimates of higher-order derivatives of holomorphic maps of the unit disk into $C_\varepsilon$. Most of this talk will be a part of on-going joint research with Seong-A Kim.
Quasiconformal maps and the ideal boundary of a Riemann surface

Masahiko Taniguchi
(Kyoto University)

It is well-known that quasiconformal maps of a Riemann surface to another can be extended to a homeomorphism between the Royden compactifications of them. We consider the essential part of the Royden boundary, and discuss the Teichmüller space of it.

REFERENCES

The essential norm of a weighted composition operator on Bergman type space in several variables

Seiichiro Ueki
(Nippon Institute of Technology)

In this talk, we estimate the essential norm of a weighted composition operator on the Bergman type space defined in the complex $n$-dimensional Euclidean space $\mathbb{C}^n$.

References

On Mobius invariant QK spaces

Hasi Wulan
(Shantou University)

We give a general theory of QK spaces of functions analytic in the unit disk, including the relationship between QK and some well-known function spaces. This talk contains some results about QK spaces in terms of Carleson type measures, boundary values, inner factors and the higher order derivatives. The compactness of composition operator from the Bloch space to QK is also given.
The integral operator with the closed range

Rikio Yoneda
(Otaru University of Commerce)

Let $g$ be an analytic function on the open unit disk $D$ in the complex plane $\mathbb{C}$. We study the following integral operators

$$J_g(f)(z) := \int_0^z f(\zeta)g'(\zeta)\,d\zeta,$$

$$I_g(f)(z) := \int_0^z f'(\zeta)g(\zeta)\,d\zeta$$

on weighted Bloch space and weighted Dirichlet spaces. Then we study the result with respect to "When do the integral operators have the closed range?"

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Quantum Douglas Algebras–Toeplitz algebras

Dechao Zheng
(Vanderbilt University)

Let $B$ be a Douglas algebra and let $\mathcal{B}$ be the algebra on the disk generated by the harmonic extensions of the functions in $B$. First we will show that the theorem analogous to Chang-Marshall theorem holds for the disk algebra $\mathcal{B}$.

For each $\alpha > -1$, the “quantum Douglas algebra” $B_\alpha$ is the Toeplitz algebra generated by Toeplitz operators (on the weighted Bergmanspace $L^2_\alpha((1-|z|^2)^\alpha dA(z)))$ with symbols in $B$.

We will show that the quantum Douglas algebra $B_\alpha$ has a canonical decomposition $S = T_{B_\alpha}S + R$ for some $R$ in the commutator ideal $C_{B_\alpha}$; and $S$ is in $C_{B_\alpha}$ iff the Berezin transform $B_\alpha S$ vanishes identically on the union of the maximal ideal space of the Douglas algebra $B$ and the set $M_1$ of trivial Gleason parts. This extends the McDonald-Sundberg Theorem and answers a question of Davidson and Douglas. Some of results are my recent joint work with S. Axler.